At the time of the computation, cumulative decimal-digit counts for $D = 10^3(10^3)10^5$ were tabulated, and nothing unexpected was observed. The final counts for $e - 2$ and $\pi - 3$ are as follows.

	0	1	2	3	4
e	9885	10264	9855	10035	10039
π	9999	10137	9908	10025	9971

	5	6	7	8	9
e	10034	10183	9875	9967	9863
π	10026	10029	10025	9978	9902

AUTHORS' SUMMARY

1. DANIEL SHANKS & JOHN W. WRENCH, JR., "Calculation of π to 100,000 decimals," *Math. Comp.*, v. 16, 1962, pp. 76–99.

47[7].—FREDERIC B. FULLER, *Tables for Continuously Iterating the Exponential and Logarithm*, ms. of 30 typewritten pages, 29 cm. Deposited in UMT file.

The theory of the continuous iteration of real functions of a real variable has been presented by a number of writers, including Bennett [1], Ward [2], and the present author [3].

The unique tables under review give 6D values of the continuously iterated function $F(x)$ and its inverse $G(x)$ for $x = 0(0.001)1$, with first differences, and for $x = 1(0.1)3$, without differences. Here $F(x)$ represents the exponential of zero iterated x times. Typical values for integral values of x are $F(0) = 0$, $F(1) = 1$, $F(2) = e$, and $F(3) = e^e$.

An introduction of five pages provides details of the procedures followed in the calculation of these tables. Appended notes explain how the tables can be extended in both directions with respect to the argument and include a discussion of the effect of the F operator on the number system of algebra.

It seems appropriate to mention here a similar study of Zavrotsky [4], which, however, led to radically different tables.

J. W. W.

1. A. A. BENNETT, "Note on an operation of the third grade," *Ann. of Math.*, v. 17, 1915–1916, pp. 74–75.
2. MORGAN WARD, "Note on the iteration of functions of one variable," *Bull. Amer. Math. Soc.*, v. 40, 1934, pp. 688–690.
3. MORGAN WARD & F. B. FULLER, "The continuous iteration of real functions," *Bull. Amer. Math. Soc.*, v. 42, 1936, pp. 393–396.
4. A. ZAVROTSKY, "Construccion de una escala continua de las operaciones aritmeticas," *Revista Ciencia e Ingeniería de la Facultad de Ingeniería de la Universidad de los Andes*, Mérida, Venezuela, December 1960, No. 7, pp. 38–53. (See *Math. Comp.*, v. 15, 1961, pp. 299–300, RMT **63.**)

48[8].—JOHN R. WOLBERG, *Prediction Analysis*, D. Van Nostrand Co., Princeton, N. J., 1967, xi + 291 pp., 24 cm. Price $10.75.

The author develops a general statistical approach, called prediction analysis, for designing experiments to estimate the structural parameters in functional relationships and regression surfaces by the general method of least squares. Structural parameters are the constants that link the true values of the independent and dependent variables. Prediction analysis considers such design problems as how to choose sample size and values of the independent variables to keep the predicted standard errors of the parameters less than a given upper bound. Treatment of these design problems in book form for scientists and engineers is long overdue. Mathematical prerequisites are elementary calculus and some exposure to matrix theory.

After a short introductory chapter defining the scope of prediction analysis, the statistical concepts essential for the remainder of the text are reviewed in chapter 2. Then, the method of least squares is developed in chapter 3 in sufficient generality to fit general nonlinear relationships to experimental data with measurement errors in the independent variables. Chapter 4 describes prediction analysis and its implementation on a computer. The author shows, for a proposed experiment, how to obtain the predicted variances for the least squares estimators of the structural parameters in a functional relationship when the measurement error variances and the form of the functional relationships are known and some knowledge is available about the structural parameters. Such predicted variances are the starting point from which an investigator would choose an experimental design. Examples to illustrate the usefulness of applying prediction analysis to the design of experiments to fit the polynomial, exponential, sine series and Gaussian function comprise chapters 5–8, respectively. Each chapter in the text concludes with a summary of the important topics, problems for solution and computer projects.

The author is only partially successful in carrying out his objective to provide a useful reference work for experimenters to design and/or analyze experiments to estimate parameters in functional relationships. Several reasons account for this assessment; only the most important difficulties will be outlined here. First, the iterative least squares procedure described in chapter 3, pages 39–45 to minimize the sum of squares in Equation 3.3.7 will not in general produce either consistent or minimum variance estimators of the structural parameters, contrary to the author's statements on pages 30, 31. The problem lies both with the quantity that is minimized and with the iterative procedure itself. Minimization of the defined sum of squares does not give the "best" estimators of the structural parameters when the measurement errors are not Gaussian. On the other hand, if the measurement errors have known variances and have a Gaussian distribution, minimization of the defined sum of squares will give the "best" estimators asymptotically; however, the iterative procedure will not yield such "best" estimators in general. In either situation when nonnegligible measurement error exists in the independent variables, the iterative procedure yields inconsistent estimators for at least some of the structural parameters in functional relationships nonlinear in the independent variables or the structural parameters. This difficulty with the iterative method means that the author's examples in chapters 5–8 need to be reworked to take the bias terms into account when they arise.

In addition to this first criticism, other important difficulties mar the text. Some remarks should have been made about minimization methods that supplement or are alternative to the iterative procedure described by the author. Next, if the distributions of the measurement errors are not Gaussian, then the estimated standard errors are not unbiased estimators of the population standard errors and the confidence intervals given in the text may produce misleading inferences. No discussion of closely related work on the design of experiments to estimate functional or regression relationships by Hoel, Kiefer and Wolfowitz and others is given or referenced. Finally, prediction analysis may be easily reformulated so that it is not necessarily based on the iterative least squares method and can make use of partial knowledge about the structural parameter and error variances in a formal way. Such a reformulation is possible through Bayesian decision theory.

To sum up, this text is a contribution to the statistical design and analysis of experiments to estimate the structural parameters in functional relationship and regression surfaces of known form. The detailed techniques developed here are reasonable to use in choosing the number of observations to take on the dependent variable and in estimating the structural parameters when the values of the independent variables are chosen in advance, the independent variables are measured without error, the variance of the measurement error for the dependent variable is known, some knowledge is available about the structural parameters and the probability distribution of the measurement error is "approximately" Gaussian. When one or more of these assumptions is not true, use of the iterative least squares method needs to be justified in each particular application.

R. M. Elashoff

University of California
San Francisco Medical Center
Computer Center
San Francisco, California

49[9].—M. Lal & P. Gillard, *Table of Euler's Phi Function, $n < 10^5$*, Memorial University of Newfoundland, St. John's, Newfoundland, October 1968, 200 pp., paperbound. Deposited in the UMT file.

The number-theoretic function $\phi(n)$ is listed for $n = 0(1)99999$, 500 values per page. If

$$n = \prod_i p_i^{a_i},$$

then

$$\phi(n) = n \prod_i \frac{p_i - 1}{p_i}, \qquad (1)$$

and the function was computed here by (1).

Earlier well-known tables of $\phi(n)$ were by J. J. Sylvester [1] (to $n = 10^3$) and J. W. L. Glaisher [2] (to $n = 10^4$). Both of these earlier authors seemed primarily interested not in $\phi(n)$, as such, but rather in $\sum \phi(n)$ and in the inverse of $\phi(n)$. In the present case, the interest seems to be in finding solutions of

$$\phi(n) = \phi(n + 1)$$

and similar functional equations.